PA-67-NA-1 CFR
31 March 1967

FINAL REPORT

SYSTEM CONFIGURATION SYNTHESIS

Contract NAS 8-20305

Prepared for

George C. Marshall Space Flight Center
Huntsville, Alabama

Period of Performance
1 April 1966 to 1 April 1967

PHOENIX ASSOCIATES INC., 1211 Stewart Ave., Bethpage, N. Y.

TABLE OF CONTENTS

TABLE OF CONTENTS (continued)

FOREWORD

This report was prepared by Phoenix Associates Incorporated, Bethpage, New York for the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, under NASA Contract NAS 8-20305. The work was administered under the direction of the Dynamic Analysis Branch, Aeroballistics Division, NASA-MSFC with Mr. Mario H. Rheinfurth as Technical Director.

This report covers work conducted from 1 April 1966 to 1 April 1967 under the direction of Mr. Nicholas C. Szuchy, the principal investigator and Miss Janet Guthrie.

ABSTRACT

This report presents a method for searching through a particular defined ordered space for that subset of systems satisfying the given requirements. It has resulted in the definition of a generalized technique for synthesizing systems using network concepts to structure the problem and the digital computer for calculating the element values. This concept is a direct approach to the optimization problem for it requires the enumeration of all possible systems within the ordered space satisfying the specifications. The optimum configuration, dependent upon the criteria, is then selected from among the calculated systems.

SECTION I

INTRODUCTION

The synthesis of systems has classically been an undertaking in which there is much art as well as science. For any given set of requirements, vastly different systems may be devised that provide a good fit within a defined tolerable error. The particular system proposed by a designer may be somewhat fortuitous, in that it is not within human capability to examine even a significant fraction of those systems which might prove suitable. Thus the need for a method of system synthesis that is suitable for machine programming as shown in Figure I-1 rests on the requirement for assurance that the end result may reasonably be called optimum.

The prime question in attempting to devise such a method of synthesis is: "In what manner may an order be imposed on the topological region containing all possible solutions, such that the development of the system may be undertaken according to logical rules?" Thus the first phase of work under this contract was an investigation to define a limited space of this region in such a way that operations within the space could be developed that are suitable for expanded use.

Considerations on the analysis and/or synthesis of systems leads naturally to the geometry of the interconnection pattern between its various discrete subsystems. These definable characteristics which constitute the system, readily lend themselves to matrix formulation and solution by digital computers; the system abstraction is definable both graphically and algebraically. In general then, the problem of synthesizing a linear system may be characterized as one of determining the connection geometry for a collection of discrete subsystems that will satisfy requirements determined: a) experimentally, by a set of observed values, or b) analytically by a system of integral-differential equations or a transfer function. Stated another way, system synthesis is the determination of a particular subset of configurations from that set which satisfies the requirements. This non-unique aspect of synthesis as shown in Figure I-2 allows for the determination of the satisfying set from yet a still larget set contained in the ordered region of all possible definable interconnection patterns of interest. Attention was confined to the two dimensional region shown in Figure I-2. That such a space was not unduly restrictive is

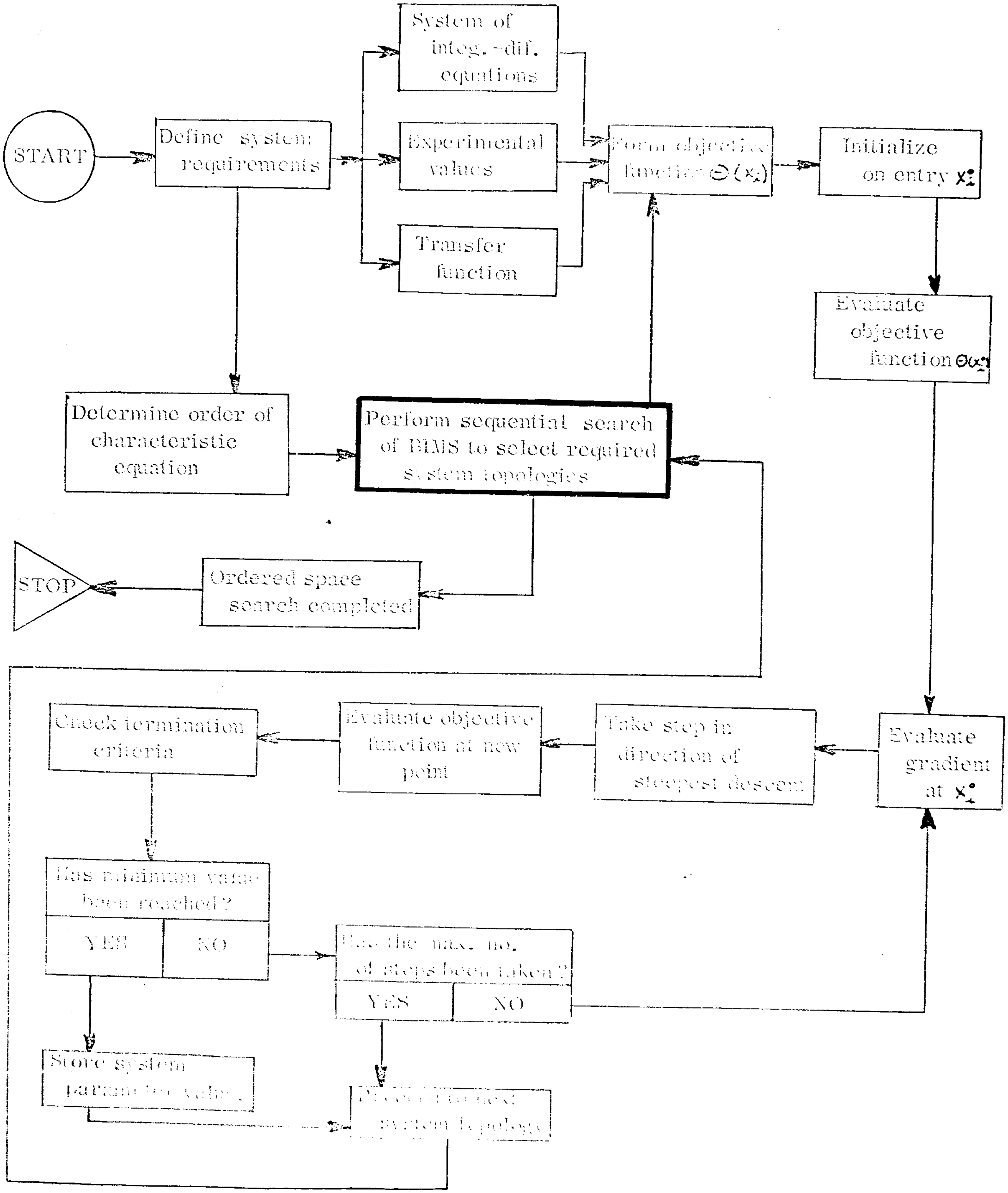


FIGURE 1. Flow Diagram for Universal System Design Procedure

indicated by the number of distinct arrangements of components that are possible within this space: 8,388,607. Upon imposing constraints such that only physically realizable systems were produced an estimated 4000 distinct patterns were formed according to machine-programable rules. From these, 110 topological configurations were determined to be basic, i.e., non-reducible and non-redundant. Within the space, systems having up to an 18^{th} order characteristic equation may be synthesized.

In summary then, this report presents a method of using those system characteristics that are definable graphically and algebraically, yet solveable by digital computer techniques, to produce a universal system synthesis (design) procedure.

The synthesis procedure, as shown in Figure I-1, begins when, knowing the order of the characteristic equation, the Basic Incidence Matrix Summary (BIMS) chart which is stored in the computer memory is entered, either at the simplest network configuration or at a higher-order configuration as required by the system. The parameters of the selected topology are then fitted to the requirements by a suitable minimization technique. If after a reasonable number of iterations the termination criteria remain unsatisfied, a step is taken to the next topological configuration. If the termination criteria are satisfied, it indicates that a system topological configuration with parameters evaluated has been determined satisfying the requirements.

Since the method is quite general it may be applied to the analysis/synthesis in any or several areas of engineering design. By the repeated application of the iterative procedures large systems having many interfaces may be developed on the computer with a smaller expenditure of engineering manpower. Consequently by using the digital computer to determine system configurations, optimum design choices can be made more intelligently at a lower engineering cost than at present.

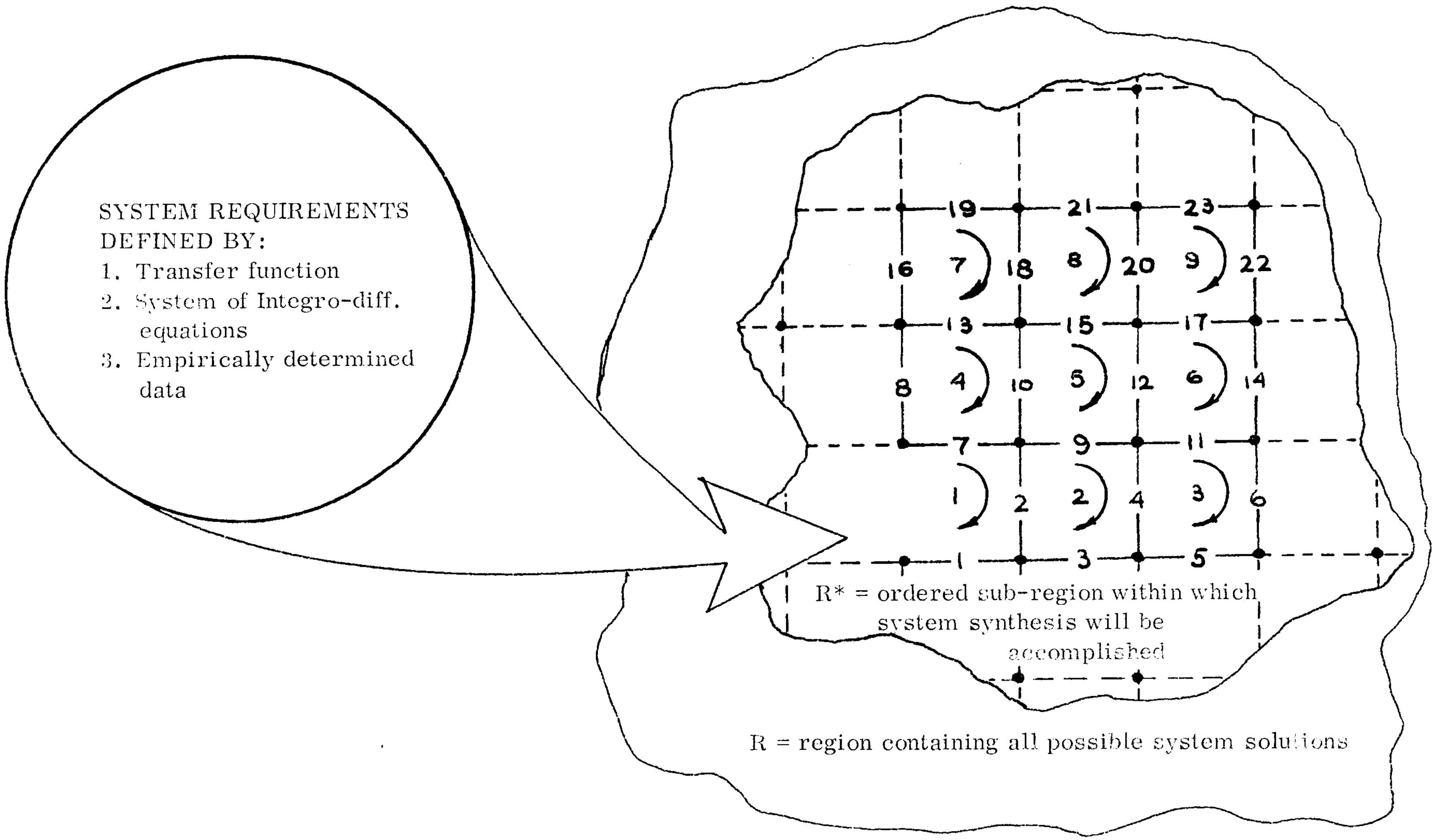


FIGURE I-2. Ordered System Configuration Search Space

SECTION II

SYSTEM NETWORK TOPOLOGY

This section develops the mathematical model for the structural or interconnection pattern of a system. The algebraic abstraction of the system is then developed in conjunction with this graphical representation.

A. SYSTEM CONCEPTS

A system as represented in Figure II-1 is defined as a collection of discrete component (subsystems) having definable characteristics which in totality constitute an entity (a system) having definable characteristics. The components of a system may be finite and physically describable, such as

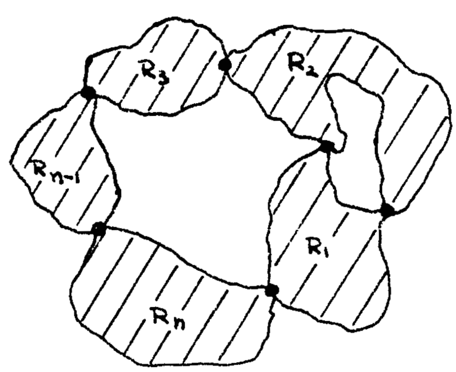


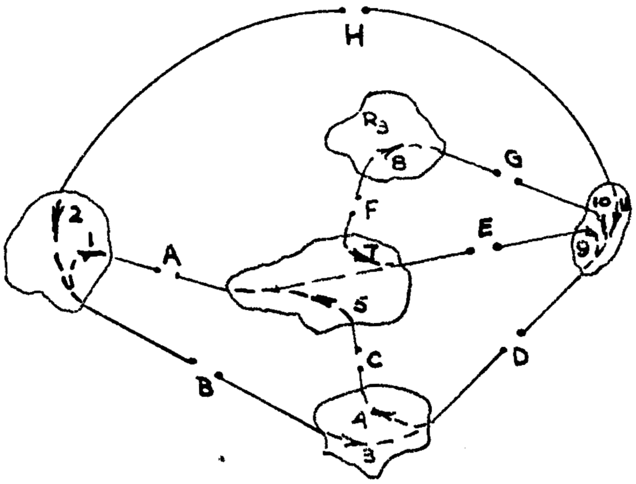
FIGURE II-1. A collection of discrete interconnected regions, forming a system.

hardware; or are less tangible discrete elements (regions) defining an associated functional interaction superimposed upon a topological structure. Figure II-1 represents diagrammatically the interconnection of the system, where the contact points are the junction or interfaces between any two regions R_{ij}, R_j.

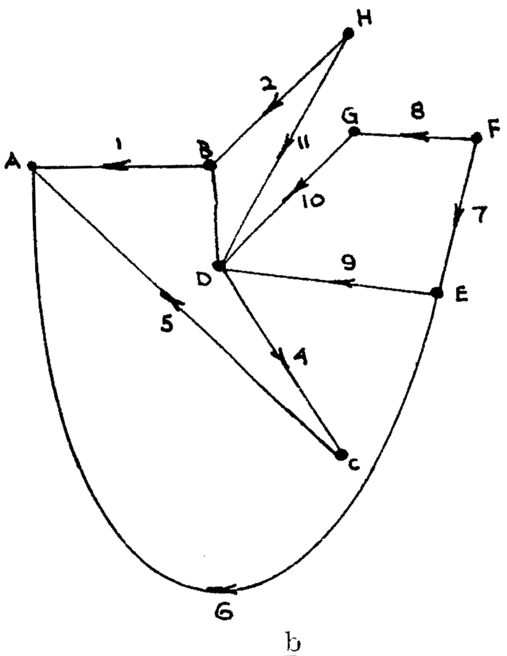
The associated system network is generated from the mathematical model by two mutually independent characteristics: (1) an algebraic structure defining the characteristics of the regions or components superimposed upon (2) the topological structure defining their interconnection pattern. This model then allows for an ordered approach to the analysis and synthesis of systems. Network concepts are applicable to a wide range of phenomena allowing for immediate generalization and applicability.

B. NETWORK AND GRAPH CONCEPTS

When considering a system from a geometric or topological point of view, its graph is of great importance. The graph of the system represents its interconnection pattern. Figure II-2a represents schematically a system with its interconnecting paths indicated, and Figure II-3b shows its corresponding graph. From the diagram it may be noted that the graph is found by replacing each of the system elements by directed lines, with each line connecting two vertices or nodes. A <u>branch</u> is a



<u>a</u>



<u>b</u>

FIGURE II-2. a) System schematic and b) its graph.

directed line segment of a graph, including its two vertices, as shown in Figure II-3. Its length or curvature has no meaning; only the nodes it connects are important. Each branch

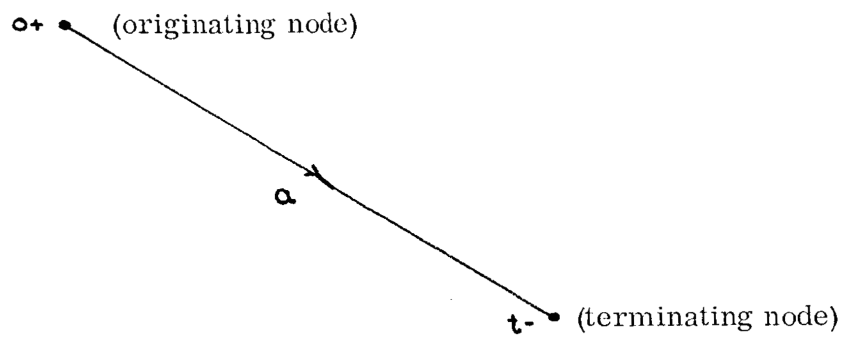


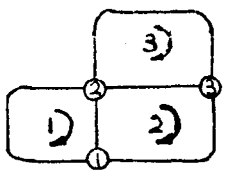
FIGURE II-3. Branch "a", showing its originating and terminating nodes.

of the network may consist of the elemental parts of the system, or a complex combination of elemental parts. In drawing a graph all energy sources are removed, with the "across" variables[1] (potential difference, relative velocity, pressure difference, etc.) being replaced by a closed path (a short - i.e., a line segment having no direction associated with it) and the "through" variables (current, force, volume flow rate, etc.) being replaced by an open path (i.c., no line segment).

Each branch in the node space set connects exactly two nodes. A particular interconnection of branches is that subset of the node space, which represents graphically the interconnection pattern of the system under consideration. To algebraically specify the incidence or connectivity relationships of the graph, a matrix is defined such that each element has the value of -1, 0, or +1. Depending upon the basic definition, two fundamental matrices are found; 1) the branch-node incidence matrix and 2) the branch-mesh incidence matrix. A one-to-one correspondence exists between the rows and columns of the matrix and the branch and the nodes or meshes of the graph. The branch-node incidence matrix A and the branch-mesh incidence matrix C are duals, and are related to each other by the fundamental relationship II-1.

$$CA^T = AC^T = 0 \qquad \text{(II-1)}$$

As a simple demonstration of the above Equation (II-1), consider the three-loop five-branch network graph shown in Figure II-3. By the appropriate matrix formulation and multiplication the null matrix is found as indicated in Equation II-1.



$$A = \begin{array}{cc|c} & & \text{BRANCHES} \\ & & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \hline \text{NODES} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & -1 & 0 & -1 & 0 \\ -1 & 1 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 \end{bmatrix} \end{array}$$

$$C = \begin{array}{cc|c} & & \text{BRANCHES} \\ & & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \hline \text{MESHES} & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \end{array}$$

FIGURE II-4. Three-loop five-branch network graph and its associated branch-node incidence matrix A and its branch-mesh incidence matrix C.

The mathematical aspects of the network graph are related through its topological properties defined in terms of the cut-set and circuit matrices. A cut-set of a particular network graph N of b branches and n nodes in the p x q node space set is a minimal subset of the branches of N whose removal separates N into exactly two connected subnetwork graphs N_s and N_s' having no common nodes. A cut-set matrix $A = (\alpha_{ib})$ is any matrix whose rows are cut-set vectors α_i.

A circuit of a particular network graph N of b branches and n nodes in the p x q node space set is a subset of branches that, together with their end points, form a simple closed curve. A circuit matrix $B = (\beta_{\upsilon b})$ is any matrix whose rows are circuit vectors β_υ.

The above two matrices lead directly to the two fundamental generalized laws of system-network graphs (generalized Kirchhoff laws):

I. The product of any circuit vector β and any across vector V is equal to zero. ($\beta V = 0$).

II. The product of any cut-set vector α and any through vector I is equal to zero ($\alpha I = 0$).

It is felt that the development of the basic topological concepts necessary for the manipulation and description of the system graphs will be enhanced by a side-by-side consideration of the particular example shown in Figure II-5.

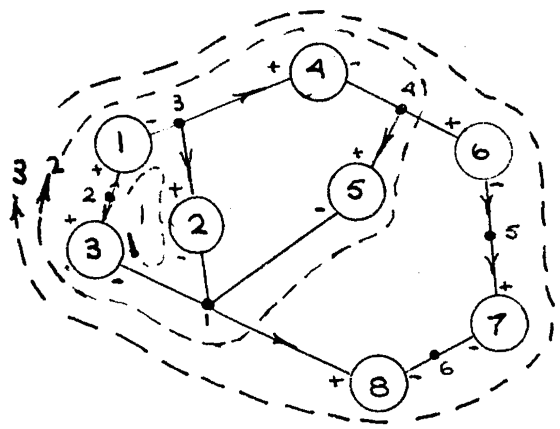


FIGURE II-5. Three-loop eight-branch network graph.

1) Generalized Kirchhoff's across relationship $\beta V = 0$

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{bmatrix} = [0] \qquad \text{(II-2)}$$

2) Generalized Kirchhoff's through relationship $\alpha I = 0$.

$$\begin{bmatrix} 0 & -1 & -1 & 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_5 \\ i_6 \\ i_7 \\ i_8 \end{bmatrix} = [0] \qquad \text{(II-3)}$$

3) The branch, through matrix is given by $I_b = \beta I_m$

$$\begin{bmatrix} i_{b_1} \\ i_{b_2} \\ i_{b_3} \\ i_{b_4} \\ i_{b_5} \\ i_{b_6} \\ i_{b_7} \\ i_{b_8} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ -1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_{m_1} \\ i_{m_2} \\ i_{m_3} \end{bmatrix} \qquad \text{(II-4)}$$

4) The branch, through matrix I_b is transformed into the across matrix V_b by pre-multiplying by the transformation branch matrix Z_{br}

$$\begin{aligned}[V_b] &= [Z_{br}]\,[I_b] \\ &= [Z_{br}]\,[\beta]\,[I_m]\end{aligned} \tag{II-5}$$

5) Defining the column matrix e_e, each of whose elements gives the total across matrix sources in a loop, the characteristic impedance of the system may now be defined as $Z = \beta Z_{br} \beta$ where Z_{br} is a diagonal matrix defining the individual branch impedances.

$$\begin{aligned}e_e &= \beta V_b \\ &= \beta Z_{br} \beta' I_m \\ &= Z I_m\end{aligned} \tag{II-6}$$

where

$$Z = \beta Z_{br} \beta' \tag{II-7}$$

6) The characteristic impedance for the particular system configuration considered may be determined by matrix multiplication as shown.

$$Z = \begin{bmatrix} (Z_1+Z_2+Z_3) & (Z_1+Z_3) & (Z_1+Z_3) \\ (Z_1+Z_3) & (Z_1+Z_4+Z_5-Z_3) & (Z_1+Z_4-Z_3) \\ (Z_1+Z_3) & (Z_1+Z_4-Z_3) & (Z_1+Z_4+Z_6+Z_7-Z_8-Z_3) \end{bmatrix} \tag{II-8}$$

C. NETWORK EQUATIONS

For a generalized n-degree of freedom linear dynamic system, the equations of motion using the Laplace transform operator may be written in the form:

$$\begin{bmatrix} F_1(s) \\ F_2(s) \\ F_3(s) \\ \vdots \\ F_n(s) \end{bmatrix} = \begin{bmatrix} Z_{11}(s) & Z_{12}(s) & Z_{13}(s) & \cdots & Z_{1n}(s) \\ Z_{21}(s) & Z_{22}(s) & Z_{23}(s) & \cdots & Z_{2n}(s) \\ Z_{31}(s) & Z_{32}(s) & Z_{33}(s) & \cdots & Z_{3n}(s) \\ \vdots & \vdots & \vdots & & \vdots \\ Z_{n1}(s) & Z_{n2}(s) & Z_{n3}(s) & \cdots & Z_{nn}(s) \end{bmatrix} \begin{bmatrix} x_1(s) \\ x_2(s) \\ x_3(s) \\ \vdots \\ x_n(s) \end{bmatrix} \tag{II-9}$$

$F_1(S), F_2(S) \ldots F_3(S)$ represent the forcing or disturbing functions applied to the generalized coordinates $X_1(S), X_2(S), \ldots X_3(S)$ with impedance functions being at most a quadratic of the form $M_{ij}S^2 + B_{ij}S + K_{ij}$, where the coefficients have positive real values (zero included). Without losing any generality, since the principle of superposition applies, only one forcing function is considered in the subsequent development, with all the others being equal to zero. The above equation may then be written as

$$[F] = [Z][X] \tag{II-10}$$

where

$$[F] = \begin{bmatrix} F_1(S) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{II-11}$$

In the frequency domain the elements Z_{ij} of $[Z]$ are in general complex impedance functions, where

$$Z_{ij} = \begin{cases} i = j & \text{represents loop self-impedance functions} \\ i \neq j & \text{represents loop mutual-impedance functions.} \end{cases} \tag{II-12}$$

Therefore, the response of the K^{th} generalized coordinate may be determined in the S-plane as:

$$x_k = \frac{|M| F_1}{|Z|} \qquad \text{(II-13)}$$

and the forcing function contribution across an impedance in the K^{th} loop is:

$$f_k = Z_k x_k \qquad \text{(II-14)}$$

resulting in a transfer function:

$$\frac{f_k}{F_1} = T(s) = \frac{|M| Z_k}{|Z|} \qquad \text{(II-15)}$$

where $|Z|$ is the determinant of the characteristic equation of the system and $|M|$ is its minor, formed by cancelling that column which belongs to the generalized coordinate of interest, and that row which corresponds in Equation (II-10) to the expression with the non-zero left side.

The determinant of the characteristic impedance developed in Equation (II-7) is identical to the one in Equation (II-13). Therefore the development of the impedance function from the network-topological point of view yields the same result as the development from a system of integro-differential equations. The admissible branch impedance considered in the development from the topological aspects is at most a quadratic in form for the diagonal matrix $[Z_{br}]$

D. SYSTEM ALGEBRA (MANIPULATION)

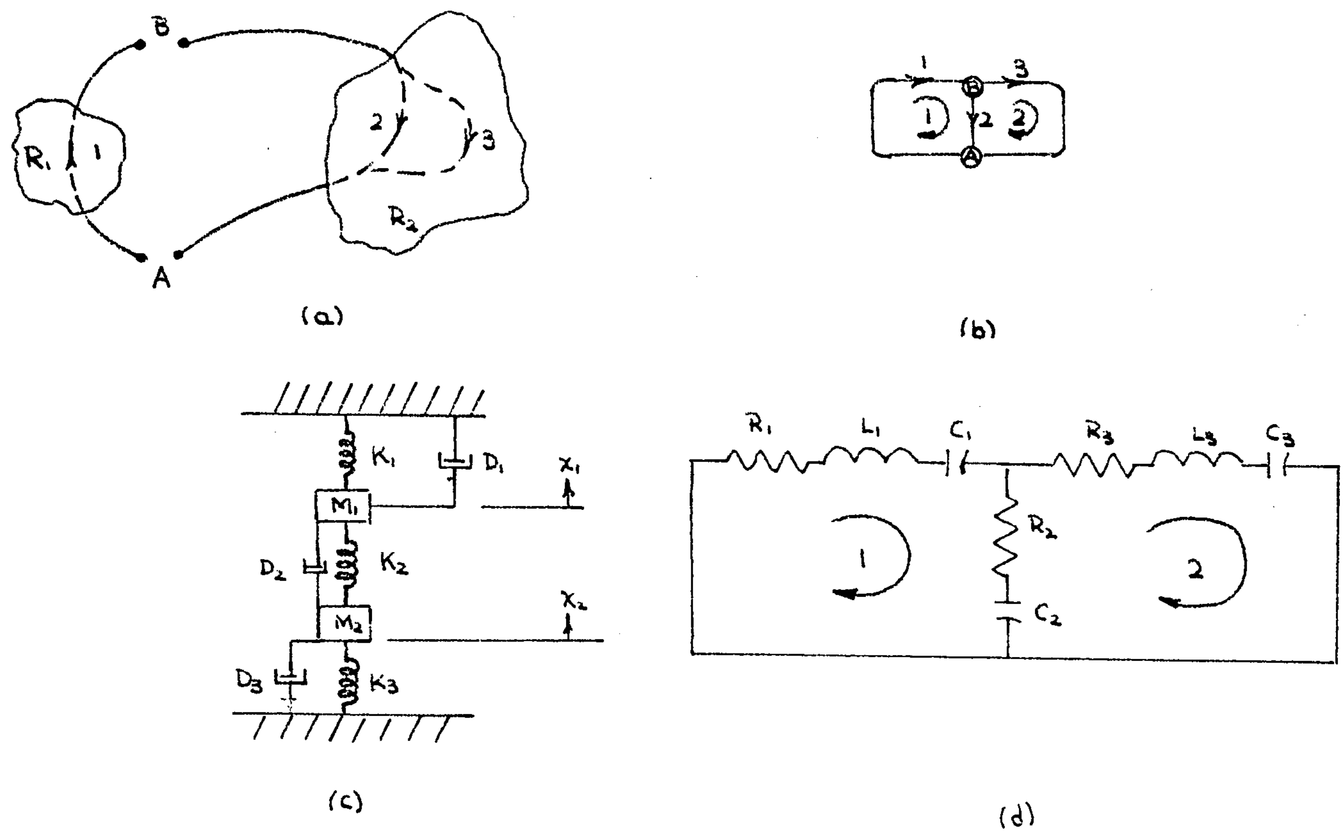


FIGURE II-6. System schematic, its graph and mechanical and electrical equivalents.

Figure II-6 shows four different system representations having the same characteristic as Equation II-16.

$$\begin{bmatrix} (M_1S^2+D_1S+K_1+D_2S+K_2) & -(D_2S+K_2) \\ -(D_2S+K_2) & (M_2S^2+D_2S+K_2+D_3S+K_3) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} (Z_1+Z_2) & -Z_2 \\ -Z_2 & (Z_2+Z_3) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{(II-16)}$$

The incidence or connectivity relationships in matrix form for the system are

a) branch-node

$$A = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \tag{II-17}$$

b) branch-mesh

$$C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \tag{II-18}$$

Since Equations II-17 and II-18 are duals of each other it may be noted that

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \tag{II-19}$$

Let it now be assumed that the system topology shown in Figure II-6b is a subset of a slightly larger topology consisting of 3-meshes and 5-branches as shown in Figure II-7.

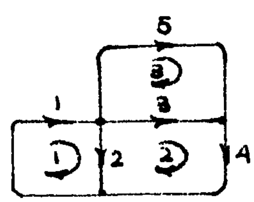


Figure II-7. 3-mesh 5-branch topology.

The incidence matrix for this particular network is given by Equation II-20

$$C = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \tag{II-20}$$

It is now possible to determine the particular topological subset shown in Figure II-6b from the larger topology in two different ways as shown in Figure II-8a and b.

FIGURE II-8. Two topological subsets of a 3-mesh 5-branch topology.

As may be noted from Figure II-8a that the network of interest contains branches 1, 2 and 3 and meshes 1 and 2, therefore the incidence matrix describing this configuration may be formed by collapsing the general rectangular incidence array by multiplying the columns containing branches 4 and 5 and row containing mesh 3 by zero to form Equation II-18. On the other hand, if the network of interest is as shown in Figure II-8b, the branches are 1, 2 and 5 and mesh 1 and another mesh generated by combining meshes 2 and 3, therefore the incidence matrix describing this configuration may be generated by collapsing the general incidence array by multiplying columns containing branches 3 and 4 by zero and adding rows 2 and 3 to again form equation II-18.

The characteristic impedance is found by matrix multiplication as shown in Equation II-21 which is identical to Equation II-16.

$$Z = C Z_{br} C^T = \begin{bmatrix} (Z_1 + Z_2) & -Z_2 \\ -Z_2 & (Z_2 + Z_5) \end{bmatrix} \qquad \text{(II-21)}$$

In the more generalized concept it is readily seen that both configurations are feasible in generating the required 2 mesh, 3-branch topology, and therefore to prevent redundant operations it is necessary to define one of these (preferably II-8a) as a basic configuration (subset) and exclude the other from any further consideration resulting in its entry in the Basic Incidence Matrix Summary (Search) chart.

SECTION III

ORDERED SPACE

The concept of the ordered topological space as presented in the previous Sections I and II introduces the associated task of defining the appropriate search mode through this space for determining topologies satisfying a transfer function of the general form:

$$T(s) = \frac{\sum_{i=0}^{n} N_i s^i}{\sum_{j=0}^{m} D_j s^j} \qquad (i<j) \qquad \text{(III-1)}$$

This section considers in detail the development of the Basic Topologies from the more general one defined in Figure III-1, and defines the required search pattern through the space.

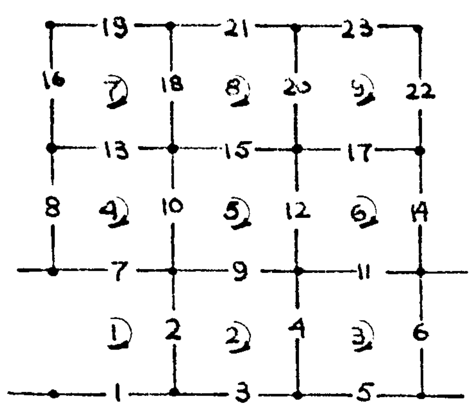


FIGURE III-1. Ordered 3 x 3 node space.

A. TOPOLOGICAL CONFIGURATION

When considering a system synthesis problem it is important to define an ordered region in which to search for those interconnection patterns that will provide acceptable structures for interpretation. As noted in Section I, three (3) loop by three (3) loop space was conceived as a region of more than adequate complexity to demonstrate the approach on small systems. As shown in Figure III-1, there is a definite order and pattern associated with both the vertical and horizontal branches, and also with the loops. The horizontal branches are all defined as odd numbered branches, while the vertical branches are defined as even numbered branches.

The general concept of the ordered space defines a problem unique to itself: Determine an optimum search path through this space such that a topology satisfying Equation III-1 may be found. Recall that any topological subset formed from the 3 x 3 node space that satisfies Equation III-1 is a solution to the system synthesis problem. Any topological subset which satisfies the laws of formation is called a <u>possible topological configuration</u>. Any possible topological configuration which satisfies the restrictions imposed by the particular node space being considered is called a <u>feasible topological configuration</u>.

Any feasible topological configuration which is a fundamental (unique) subset of the particular node space being considered, and to which other feasible configurations may be reduced, is called a basic topological configuration. The search mode through the space then, is defined in terms of an ordered progression through the basic topological configurations. Table III-1 summarizes the 96 forms of mesh combination for generating the feasible topological configurations.

B. LAWS OF FORMATION

Considering the specific 3 x 3 node space previously defined in Figure III-1, the laws of formation for the possible topological configurations up to and including 3 loop networks, are formulated using the following logic symbols and notations:

M_a = represents the meshes associated with node space (for a 3 x 3, a = 1,2,3...,9)

L_b = represents the various loops of interest associated with the node space, and consists of one or more meshes

$\oplus$ = logic symbol representing "or"

$*$ = logic symbol representing "and"

$(\overline{\ \ })$ = logic symbol representing "not"

$\sum_{n=\alpha}^{\gamma}$ = symbol used to indicate more than one combined "or" operation

$\prod_{n=\alpha}^{\gamma}$ = symbol used to indicate more than one combined "and" operation.

TABLE III-1

POSSIBLE MESH COMBINATIONS		
No. of Loops	Possible Mesh Combinations	Mesh Combinations
1	9	1,2,3,4,5,6,7,8,9
2	20	(11,12,13,141,15,16,17,18) (22,23,24,25,26,27) (33,34,35,36) (44,45)
3	23	(111,112,113,114,115,116,117) (122,123,124,125,126) (133,134,135) (144) (222,223,224,225) (233,234) (333)
4	18	(1111,1112,1113,1114,1115,1116) (1122,1123,1124,1125) (1133,1134) (1222,1223,1224) (1233) (2222,2223)
5	12	(11111,11112,11113,11114,11115) (11122,11123,11124) (11133) (11222,11223) (11133)
6	17	(111111,111112,111113,111114) (111122,111123) (111222)
7	4	(1111111,1111112,1111113) (1111122)
8	2	(11111111,11111112)
9	1	(111111111)

These logic laws may be programmed on the digital computer to define the basic topological space that will be used in the general search program for synthesizing the system.

<u>1 Loop</u>

$$[L_i = M_k] \ (1,2,3,4,5,6,7,8,9)$$

This represents an almost trivial case where for <u>any</u> topological configuration there exists only one basic topology $L_1 = M_1$.

<u>2 Loop</u>

$$[L_i, L_j] = [M_k, M_p M_q M_r M_s M_t M_u M_v M_w]$$

$$(|k-p| \oplus |k-q| \oplus |k-r| \oplus |k-s| \oplus |k-t| \oplus |k-u| \oplus |k-v| \oplus |k-w|) * (|p-q| \oplus |p-r| \oplus |p-s| \oplus |p-t| \oplus |p-u| \oplus |p-v| \oplus |p-w|)$$
$$* (|q-r| \oplus |q-s| \oplus |q-t| \oplus |q-u| \oplus |q-v| \oplus |q-w|) * (|r-s| \oplus |r-t| \oplus |r-u| \oplus |r-v| \oplus |r-w|)$$
$$* (|s-t| \oplus |s-u| \oplus |s-v| \oplus |s-w|) * (|t-u| \oplus |t-v| \oplus |t-w|) * (|u-v| \oplus |u-w|) * (|v-w|) = 1 \oplus 3$$

which may be written in a shortened form as

$$\left(\sum_{a=p}^{w} |k-a|\right) * \left(\sum_{b=q}^{w} |p-b|\right) * \left(\sum_{c=r}^{w} |q-c|\right) * \left(\sum_{d=s}^{w} |r-d|\right) * \left(\sum_{e=t}^{w} |s-e|\right) * \left(\sum_{f=u}^{w} |t-f|\right) * \left(\sum_{g=v}^{w} |u-g|\right) * (|v-w|) = 1 \oplus 3$$

$$[L_i, L_j] = [M_k M_p, M_q M_r M_s M_t M_u M_v M_w]$$

$$(|k-p|) * \left(\sum_{a=q}^{w} |k-a| \oplus |p-a|\right) * \left(\sum_{c=r}^{w} |q-c|\right) * \left(\sum_{d=s}^{w} |r-d|\right) * \left(\sum_{e=t}^{w} |s-e|\right) * \left(\sum_{f=u}^{w} |t-f|\right) * \left(\sum_{g=v}^{w} |u-g|\right) * (|v-w|) = 1 \oplus 3$$

$$[L_i, L_j] = [M_k M_p M_q, M_r M_s M_t M_u M_v M_w]$$

$$\left(\sum_{a=p}^{q} |k-a|\right) \oplus (|p-q|) * \left(\sum_{b=r}^{w} |k-b| \oplus |p-b| \oplus |q-b|\right) * \left(\sum_{c=s}^{w} |r-c|\right) * \left(\sum_{d=t}^{w} |s-d|\right) * \left(\sum_{e=u}^{w} |t-e|\right) * \left(\sum_{f=v}^{w} |u-f|\right) * (|v-w|) = 1 \oplus 3$$

$$[L_i, L_j] = [M_k M_p M_q M_r, M_s M_t M_u M_v M_w]$$

$$\left(\sum_{a=p}^{r} |k-a| \oplus \sum_{b=q}^{r} |p-b| \oplus |q-r|\right) * \left(\sum_{c=t}^{w} |s-c|\right) * \left(\sum_{d=u}^{w} |t-d|\right) * \left(\sum_{e=v}^{w} |u-e|\right) * (|v-w|) = 1 \oplus 3$$

3 Loop

$$[L_i, L_j, L_a] = [M_k, M_p, M_q M_r M_s M_t M_u M_v M_w]$$
$$(|k-p|) \ast \left(\sum_{a=q}^{w} |k-a| \oplus \sum_{b=q}^{w} |p-b|\right) \ast \left(\sum_{c=r}^{w} |q-c|\right) \ast \left(\sum_{d=s}^{w} |r-d|\right) \ast \left(\sum_{e=t}^{w} |s-e|\right) \ast \left(\sum_{f=u}^{w} |t-f|\right) + \left(\sum_{g=v}^{v} |u-v|\right) \ast (v-w) = 1 \oplus 3$$

$$[L_i, L_j, L_a] = [M_k, M_p M_q, M_r M_s M_t M_u M_v M_w]$$
$$(|k-p| \oplus |k-q|) \ast (|p-q|) \ast \left(\sum_{a=r}^{w} |p-a| \oplus |q-a|\right) \ast \left(\sum_{b=s}^{w} |r-b|\right) \ast \left(\sum_{c=t}^{w} |s-c|\right) \ast \left(\sum_{d=u}^{w} |t-d|\right) \ast \left(\sum_{e=v}^{w} |u-v|\right) \ast (v-w) = 1 \oplus 3$$

$$[L_i, L_j, L_a] = [M_k, M_p M_q M_r, M_s M_t M_u M_v M_w]$$
$$(|k-p| \oplus |k-q| \oplus |k-r|) \ast (|p-q| \oplus |p-r| \oplus |q-r|) \ast \left(\sum_{a=s}^{w} |p-a| \oplus |q-a| \oplus |r-a|\right) \ast \left(\sum_{b=t}^{w} |s-b|\right) \ast \left(\sum_{c=u}^{w} |s-c|\right) \ast \left(\sum_{d=v}^{w} |s-d|\right) \ast (|v-w|)$$
$$= 1 \oplus 3$$

$$[L_i, L_j, L_a] = [M_k, M_p M_q M_r M_s, M_t M_u M_v M_w]$$
$$(|k-p| \oplus |k-q| \oplus |k-r| \oplus |k-s|) \ast \left(\sum_{a=q}^{s} |p-a| \oplus \sum_{b=r}^{s} |q-b| \oplus |r-s|\right) \ast \left(\sum_{c=t}^{w} |p-c| \oplus |q-c| \oplus |r-c| \oplus |s-c|\right)$$
$$\ast \left(\sum_{d=u}^{w} |t-d|\right) \ast \left(\sum_{c=v}^{w} |u-c|\right) \ast (|v-w|) = 1 \oplus 3$$

$$[L_i, L_j, L_a] = [M_k M_p, M_q M_r, M_s M_t M_u M_v M_w]$$
$$(|k-p|) \ast (|k-q| \oplus |k-r| \oplus |p-q| \oplus |p-r|) \ast \left(\sum_{a=s}^{w} |q-a| \oplus |r-a|\right) \ast \left(\sum_{b=t}^{w} |s-b|\right) \ast \left(\sum_{c=u}^{w} |t-c|\right) \ast \left(\sum_{d=v}^{w} |u-d|\right) \ast (|v-w|) = 1 \oplus 3$$

$$[L_i, L_j, L_a] = [M_k M_p, M_q M_r M_s, M_t M_u M_v M_w]$$
$$(|k-p|) \ast \left(\sum_{a=q}^{s} |k-a| \oplus |p-a|\right) \ast \left(\sum_{b=t}^{w} |q-b| \oplus |r-b| \oplus |s-b|\right) \ast \left(\sum_{c=u}^{w} |t-c|\right) \ast \left(\sum_{d=v}^{w} |u-d|\right) \ast (|v-w|) = 1 \oplus 3$$

$$[L_i, L_j, L_a] = [M_k M_p M_q, M_r M_s M_t, M_u M_v M_w]$$
$$(|k-p| \oplus |k-q| \oplus |p-q|) \ast \left(\sum_{a=r}^{t} |k-a| \oplus |p-a| \oplus |q-a|\right) \ast (|r-s| \oplus |r-t| \oplus |s-t|) \ast \left(\sum_{b=u}^{w} |r-b| \oplus |s-b| + |s-t|\right)$$
$$\ast \left(\sum_{c=v}^{w} |u-c|\right) \ast (|v-w|) = 1 \oplus 3.$$

C. BASIC INCIDENCE MATRIX SUMMARY (BIMS)

From the larger population of configurations as generated from the particular ordered 3 x 3 node space of Figure III-1, 110 were defined as being basic configurations, for a two port three terminal topology. The Basic Incidence Matrix Summary (BIMS) chart, Figures III-2 and III-3 presents in a form readily programmable for computer application the 110 basic networks to be used sequentially in the search for systematically synthesized systems satisfying specification sets. The generalized branch-mesh incidence matrix (lower right hand corner) is stored in the computer memory banks. A particular basic incidence matrix is generated by collapsing the generalized array according to the rules defined by the column of branch entries and the row of mesh entries, as modified by the combinational laws.

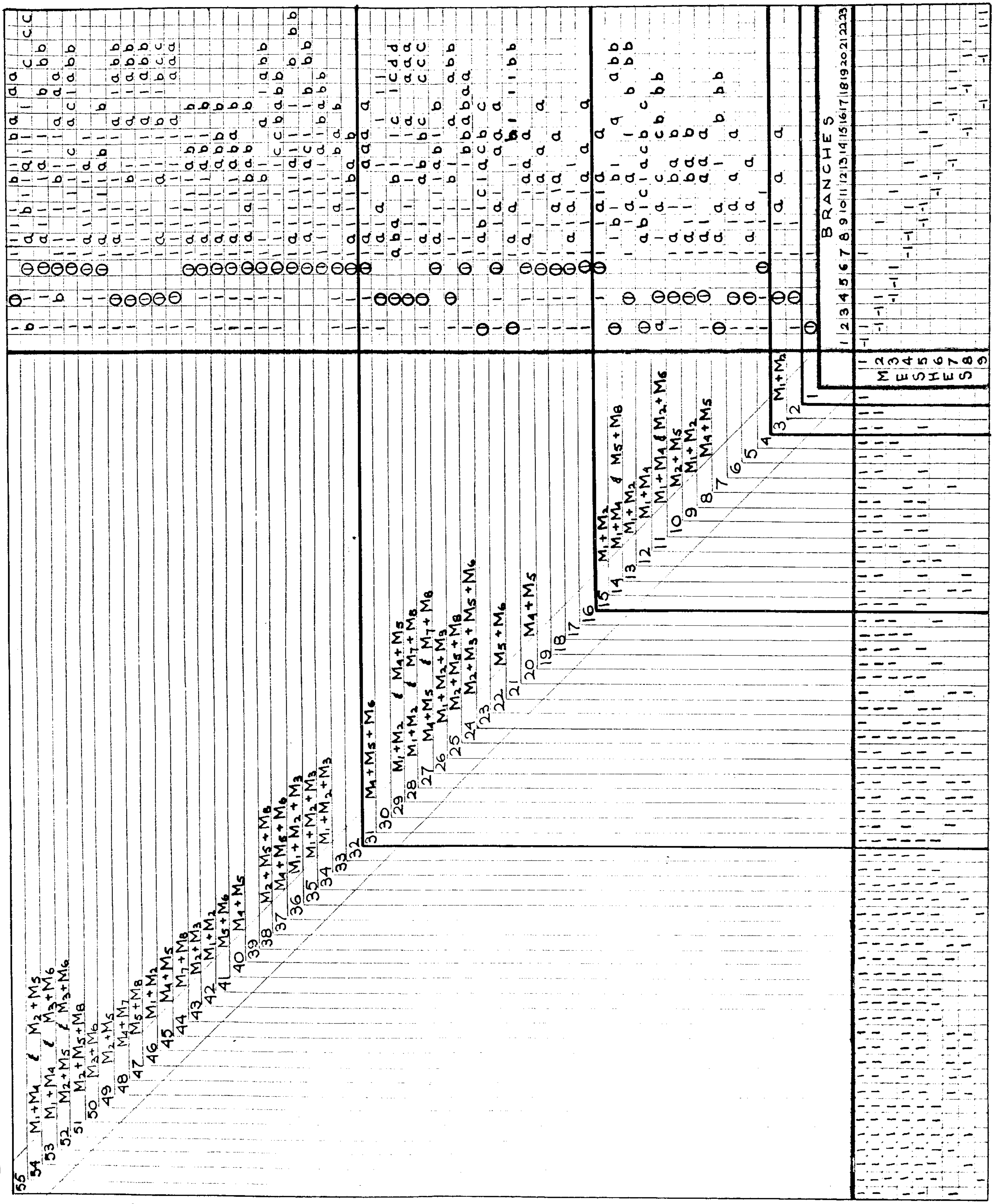


FIGURE III - 2 BIMS Chart

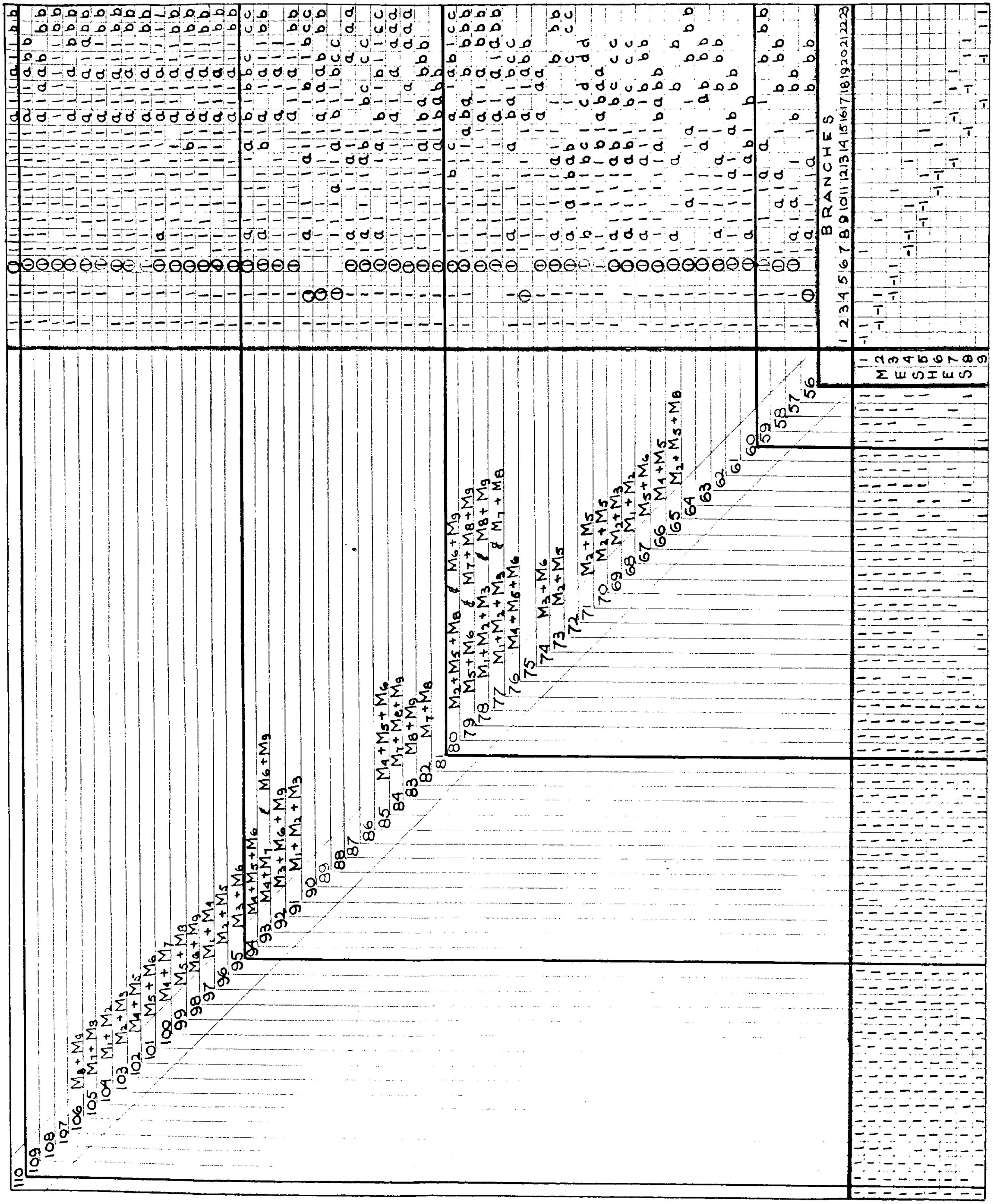


FIGURE III-3 BIMS Chart (continued)

SECTION IV

SYSTEM SYNTHESIS

This section is expository in nature, since it demonstrates the approach and techniques shown in Figure I-1 for synthesizing a particular system. It is assumed that the over-all transfer function for the required system is known and is a stable and rational function and is of the form:

$$T_G(s) = \frac{\sum_{i=0}^{n} a_i s^i}{\sum_{j=0}^{m} b_j s^j} \qquad (n \leq m) \qquad \text{(IV-1)}$$

It is also assumed that the denominator is of order m such that there exists at least one topological subset within the 3 x 3 node space that will satisfy it. It is now postulated that, a) the search through the node space is at the network configuration 8, or b) it is starting at configuration 8.

A. DEVELOPING A NODE SPACE TRANSFER FUNCTION

<u>Step 1</u>. From the BIMS chart III-2 it may be construed that it is the three loop, 5 branch interconnection configuration shown in Figure IV-1 superimposed on the 3 x 3 node space.

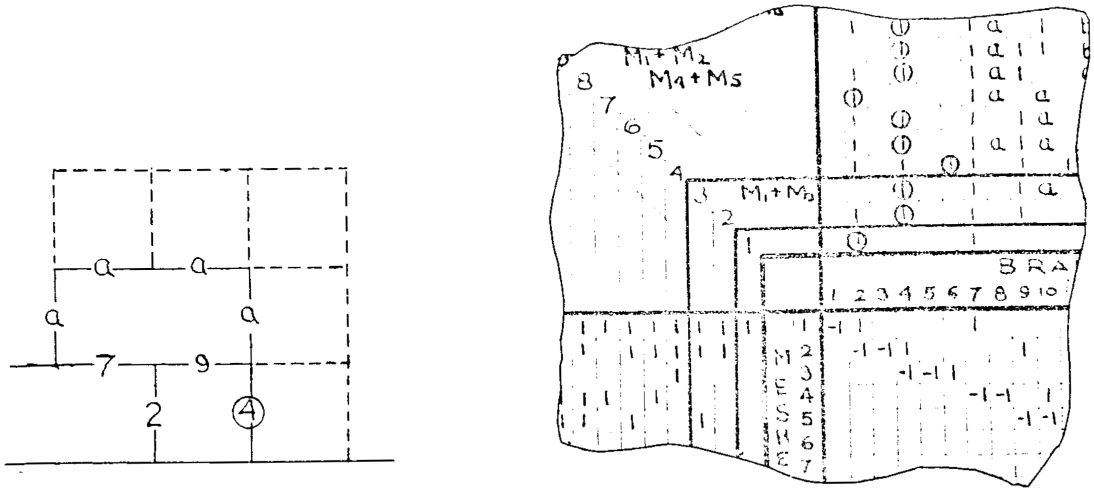


FIGURE IV-1. Interconnection configuration of basic topology number 8 and a portion of the generalized incidence matrix.

In forming the particular incidence array of interest let it be defined that the literal "a" represents branch number 8; therefore the basic topology is defined in terms of meshes 1, 2, 4 and 5 with the included branches 2, 4, 7, 8 and 9. The first stage of the collapsed incidence array for network 8 is shown in Figure IV-2 (still retaining original definitions of branches and nodes).

	2	4	7	8	9
1	1		1		
2	-1	1			1
4			-1	-1	
5					-1

FIGURE IV-2. First stage of defining incidence array for topology number 8.

Step 2. If two or more meshes define a loop as indicated by the symbolism $M_i + M_j + \ldots + M_K$ on the side of the table, then it is necessary to add the required rows of the array to form the particular loops of interest. In this case it is necessary to add the 3^{rd} and 4^{th} rows (meshes 4 and 5) together forming a 3 x 5 matrix shown in Figure IV-3. (It is not necessary to remember any of the meshes or branches because they

	2	4	7	8	9
1	1		1		
2	-1	1			1
4+5			-1	-1	-1

FIGURE IV-3. Second stage of defining incidence matrix for topology number 8.

are unique to the 3 x 3 node space and can always be determined). If two or more meshes are not combined to form a loop - Step 2 is not required.

Step 3. The characteristic equation (impedance) of the system can be determined by matrix multiplication.

$$Z = \mathrm{J}\, Z_p \mathrm{J}^T \qquad \text{(IV-2)}$$

where

$$Z = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} Z_2 & & & & \\ & Z_4 & & & \\ & & Z_7 & & \\ & & & Z_8 & \\ & & & & Z_9 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix} \qquad \text{(IV-3)}$$

resulting in

$$Z = \begin{bmatrix} (Z_2+Z_7) & -Z_2 & -Z_7 \\ -Z_2 & (Z_2+Z_4+Z_9) & -Z_9 \\ -Z_7 & -Z_9 & (Z_7+Z_8+Z_9) \end{bmatrix} \qquad \text{(IV-4)}$$

Step 4. To evaluate the matrix and determine the transfer function of the network, form the expression for the transfer function

$$T_N(s) = \frac{|M| \; Z_L}{|Z|} \qquad \text{(IV-5)}$$

where $|M|$ represents the minor of the determinant $|Z|$, resulting from the cancellation of that column which belongs to the generalized coordinate of interest, and of that row which corresponds to the particular forcing function of interest.

$$T_N(S) = \frac{\begin{vmatrix} -Z_2 & -Z_9 \\ -Z_7 & (Z_7+Z_8+Z_9) \end{vmatrix} \; Z_4}{\begin{vmatrix} (Z_2+Z_7) & -Z_2 & -Z_7 \\ -Z_2 & (Z_2+Z_4+Z_9) & -Z_9 \\ -Z_7 & -Z_9 & (Z_7+Z_8+Z_9) \end{vmatrix}} \qquad \text{(IV-6)}$$

Step 5. Form the objective function (here, again, adduced arbitrarily

$$\theta(\omega) = \left|\left(\frac{T_G(\omega) - T_N(\omega)}{T_G(\omega)}\right)\right|^2 \qquad \text{(IV-7)}$$

Step 6. Initialize the objective function $\theta(\omega)$ by postulating a set of positive values for the coefficients of $T_N(\omega)$. One such choice of values might be unity for each, as an initial step.

Step 7. Proceed to minimize the objective function $\theta(\omega)$ by the gradient technique - continue iteration process until the prescribed minimum is determined or until the appropriate number of iterations have been completed. If a minimum is reached then this topology with the calculated values represents a possible system configuration. If on the other hand a satisfactory minimum was not reached after a predetermined number of iterations, proceed to basic topology number 9 and proceed in the manner outlined above. This process is repeated a) until a system configuration is synthesized or b) until all the possible system configurations for the specified region have been determined.

SECTION V

NETWORK TOPOLOGY - MECHANICAL IMPEDANCE SYSTEM

This section outlines the procedure for writing the equations of motion of a coupled mechanical system in terms of its network parameters. The relationships are developed from Lagrange's equations with electro-mechanical analogs and impedance concepts.

Mechanical-electrical network analogies are founded on a comparison of Lagrange's equations for two similar systems. The necessary quantities are the potential and kinetic energy stored in the two systems and the dissipation, expressed as functions of a generalized coordinate which in a mechanical system is the displacement and in an electrical system is the current.

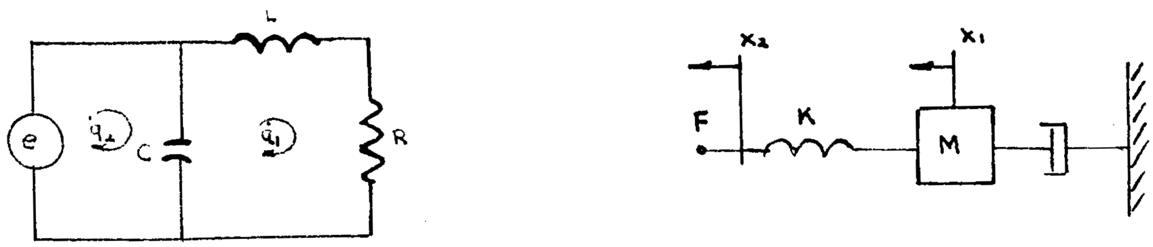


FIGURE V-1

Given the two simple systems shown in Figure V-1, the following relationships may be written:

	Electrical	Mechanical	
Kinetic Energy	$T = \frac{1}{2} L \dot{q}_1^2$	$T = \frac{1}{2} M \dot{x}_1^2$	(V-1)
Potential Energy	$V = \frac{1}{2} \frac{1}{C} (q_2 - q_1)^2$	$V = \frac{1}{2} K (x_2 - x_1)^2$	(V-2)
Dissipation	$D = R \dot{q}_1^2$	$D = R \dot{x}_1^2$	(V-3)

Lagrange's equations for the generalized coordinates are:

$$\frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{q}_n}\right) - \frac{\partial (T-V)}{\partial q_n} + \frac{1}{2}\frac{\partial D}{\partial \dot{q}_n} = e_n \qquad \frac{\partial}{\partial t}\left(\frac{\partial T}{\partial \dot{x}_n}\right) - \frac{\partial (T-V)}{\partial x_n} + \frac{1}{2}\frac{\partial D}{\partial \dot{x}_n} = f_n \qquad \text{(V-4)}$$

Expanding the equations for each of these systems,

$$\frac{\partial}{\partial t}\left(\frac{\partial (\frac{1}{2} L \dot{q}_1^2)}{\partial \dot{q}_1}\right) - \frac{\partial}{\partial q_1}\left(-\frac{1}{2}\frac{1}{C}[q_2 - q_1]^2\right) + \frac{1}{2}\frac{\partial}{\partial \dot{q}_1}(R \dot{q}_1^2) = 0 \qquad \frac{\partial}{\partial t}\left(\frac{\partial (\frac{1}{2} M \dot{x}_1^2)}{\partial \dot{x}_1}\right) - \frac{\partial (-\frac{1}{2} K [x_2 - x_1]^2)}{\partial x_1} + \frac{1}{2}\frac{\partial R \dot{x}_1^2}{\partial \dot{x}_1} = 0 \qquad \text{(V-5)}$$

$$\frac{\partial}{\partial t}(0) - \frac{\partial}{\partial q_2}\left(-\frac{1}{2C}(q_2 - q_1)^2\right) + 0 = 0 \qquad \frac{\partial}{\partial t}(0) - \frac{\partial}{\partial x_2}\left(-(\tfrac{1}{2} K (x_2 - x_1)^2\right) + 0 = f \qquad \text{(V-6)}$$

or

$$L \ddot{q}_1 + R \dot{q}_1 + \frac{1}{C}(q_1 - q_2) = 0 \qquad M \ddot{x}_1 + R \dot{x}_1 + K(x_1 - x_2) = 0 \qquad \text{(V-7)}$$

$$\frac{1}{c}(q_2 - q_1) = e \qquad\qquad K(x_2 - x_1) = f \qquad\qquad \text{(V-8)}$$

A comparison of the coefficients of the above equations indicates the following analogies:

voltage v	. . . . .	f force
charge q	. . . . .	x displacement
current i	. . . . .	x velocity
capacitance c	. . . . .	$\frac{1}{K}$ inverse spring constant (compliance)
inductance L	. . . . .	M mass

The corresponding impedance of primitive elements may now be written as shown in Table V-1.

TABLE V-1

PRIMITIVE ELEMENT IMPEDANCES Z(s)	
Mechanical	Electrical
$F_K = K\int \dot{x}\,dt = \frac{K\dot{x}}{s}$ $Z_K = \frac{F_K}{\dot{x}} = \frac{K}{s}$	$V_C = \frac{1}{C}\int i\,dt = \frac{i}{Cs}$ $Z_C = \frac{V_C}{i} = \frac{1}{Cs}$
$F_M = M\frac{d\dot{x}}{dt} = Ms\dot{x}$ $Z_M = \frac{F_M}{\dot{x}} = Ms$	$V_L = L\frac{di}{dt} = Lsi$ $Z_L = \frac{V_L}{i} = Ls$
$F_D = D\dot{x}$ $Z_D = \frac{F_D}{\dot{x}} = D$	$V_R = Ri$ $Z_R = \frac{V_R}{i} = R$

It should be noted that the number of generalized coordinates (the index n in Equation V-4) is the number of degrees of freedom of the system. It follows that if mechanical elements act in parallel to each other as shown in Figure V-2a; that is, if the forces contributed by them are functions of the same motion or motions, there is only one degree of freedom and the appropriate expression is

$$M\ddot{x} + D\dot{x} + Kx = 0 \qquad \text{(by Newton's law)} \quad \text{(V-9)}$$

or

$$\left(Ms + D + \frac{1}{s}K\right)\dot{x} = 0 \qquad \text{(V-10)}$$

so that the lumped impedance of mechanical elements acting in parallel is the sum of their individual impedances.

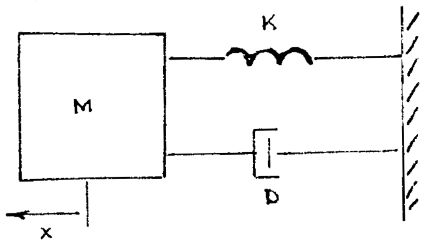


FIGURE V-2a

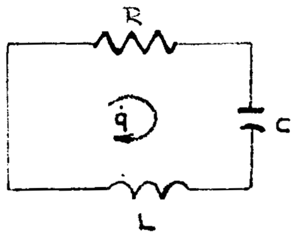


FIGURE V-2b

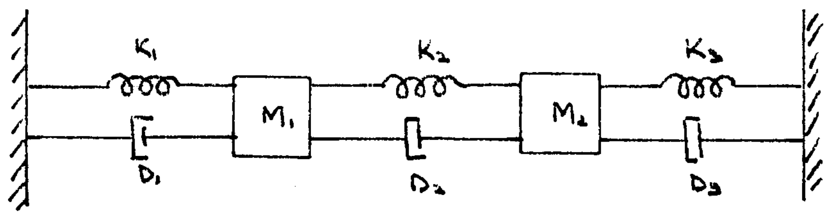
The analogous electrical circuit is shown in Figure V-2b, where the expression is

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt = 0 \qquad \text{(by Kirchoff's law)} \quad \text{(V-11)}$$

or

$$\left(Ls + R + \frac{1}{Cs}\right) i = 0 \qquad \text{(V-12)}$$

Figure V-3 shows a simple coupled mechanical system for which the equations of motion are required.



·FIGURE V-3

The two coordinates needed to describe the state of this system are X_1 and X_2, the displacements of the masses from their rest positions. From the diagram it is clear that the forces which exist on M_1 due to the prime elements K_1, D_1 and M_1 are dependent only on the motion of M_1; and similarly the forces on M_2 due to K_3,D_3 and M_2 are due only to the motion of M_2. The elements K_2 and D_2 exert forces on both M_1 and M_2 and the magnitude of these forces are dependent upon the relative motion between the two masses. These relationships suggest the possibility of representing this mechanical system in terms of the two loop network shown in Figure V-3, where Z_1 represents the equivalent mechanical impedance associated with the motion of $M_1 Z_2$ the impedance associated with the motion of M_2 and Z_C the coupling impedance associated with the relative motion between M_1 and M_2.

Returning to the example of Figure V-2, the equations for the loop velocities X_1 and X_2 can be written by application of Lagrange's equations

$$\begin{bmatrix} M_1 s + D_1 + \frac{1}{s} K_1 + D_2 + \frac{1}{s} K_2 & -D_2 - \frac{1}{s} K_2 \\ -D_2 - \frac{1}{s} K_2 & M_2 s + D_3 + \frac{1}{s} K_3 + D_2 + \frac{1}{s} K_2 \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = 0 \qquad \text{(V-13)}$$

or from the lumped-impedance equivalent of Figure V-3,

$$\begin{bmatrix} Z_1 + Z_C & -Z_C \\ -Z_C & Z_2 + Z_C \end{bmatrix} \begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = 0 \qquad \text{(V-14)}$$

An alternative is provided by noting that V-13 may be written

$$\begin{bmatrix} M_1 s^2 + D_1 s + K_1 + D_2 s + K_2 & -D_2 s - K_2 \\ -D_2 s - K_2 & M_2 s^2 + D_3 s + K_3 + D_2 s + K_2 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = 0 \qquad \text{(V-15)}$$

By re-defining the impedance as the force divided by the displacement,

$$Ms^2 = Z_M S = \frac{F_M S}{\dot{x}} = \frac{F_M}{x} = Z'_M \qquad \text{(V-16)}$$

$$Ds = Z_D S = \frac{F_D S}{\dot{x}} = \frac{F_D}{x} = Z'_D \qquad \text{(V-17)}$$

$$K = Z_K S = \frac{F_K S}{\dot{x}} = \frac{F_K}{x} = Z'_K \qquad \text{(V-18)}$$

the analogous loop can be drawn directly in terms of displacements X_1 and X_2 and impedances Z'.

In summary, the method of writing the dynamic equation of motion of a coupled mechanical system for network presentation consists of the following steps:

1) Group parallel elements into equivalent impedances by summing their individual impedances (Z'_K Z'_M Z_C)

2) Draw the analagous network diagram, being sure there is a loop and a lumped impedance for each degree of freedom, with coupling impedances for elements which are affected by two or more motions.

3) From the impedance network write the loop equations as for an electrical impedance network.

SECTION VI

STATE-VARIABLE TECHNIQUES FOR SYSTEM NETWORKS

In the synthesis of linear systems using the network approach defined in the previous sections, a possible application is the design of controllers. This section considers a transfer function of the form

$$T(S) = \frac{C(s)}{R(s)} = \frac{\Sigma a_n s^n}{\Sigma b_m s^m} \qquad (n < m)$$

to determine an applicable topological form, and develop the transfer function in terms of the state-variables of the system network.

A. CONSIDERATIONS ON THE STATE-VARIABLE METHOD FOR NETWORKS

The state-variable description of a system is a powerful technique, in which the value of all the variables which describe the system behavior at a particular instant of time t are known. The continuous sequence of points indicating the values assumed by the states is called the trajectory. A state or vector is an element which represents a property, quantity or functional relationship of something. Any system can be defined in terms of a collection of states or vectors, called a vector space.

Schematically, the "black box" shown in Figure VI-1 has a set of input, state and output variables. The number of state-variables required to specify the system uniquely is determined by the degrees of freedom, or by the degree of the characteristic

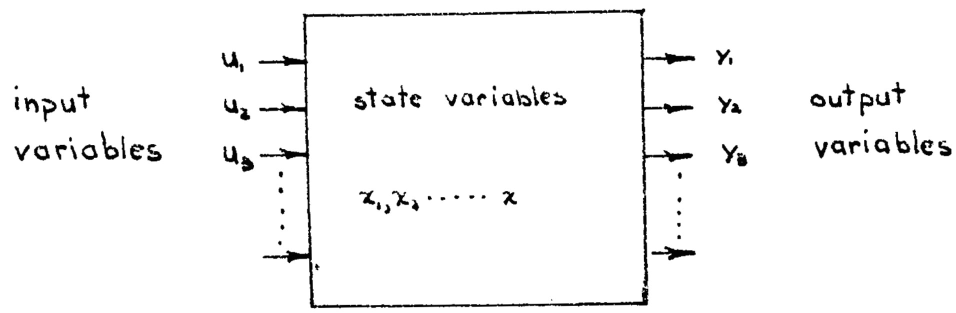


FIGURE VI-1. System schematic for state-variable representation

equation of the system. In vector notation the state may be represented as a 1 x n row matrix.

$$\underline{x}(t) = [x_1(t), x_2(t), \cdots\cdots x_n(t)] \tag{VI-1}$$

or as an n x 1 column matrix

$$\underline{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} \tag{VI-2}$$

The basic canonic equations of a system describable by a set of linear differential equations are of the form:

$$\dot{\underline{x}} = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \qquad \text{(VI-3a)}$$

$$\underline{Y} = \underline{C}\,\underline{x} + \underline{D}\,\underline{u} \qquad \text{(VI-3b)}$$

where $\underline{A}$ is the coefficient vector, $\underline{B}$ is the driving vector, $\underline{C}$ is the output vector and $\underline{D}$ is the transmission vector of the system.

<u>Example 1</u>: Given the differential equation of the system as:

$$\ddddot{v}(t) + a_3\dddot{v}(t) + a_2\ddot{v}(t) + a_1\dot{v}(t) + a_0 v(t) = b_0 u(t) \qquad \text{(VI-4)}$$

specify the state-variables:

Let	therefore
$x_1(t) = v(t)$	$\dot{x}_1(t) = x_2(t)$
$x_2(t) = \dot{v}(t)$	$\dot{x}_2(t) = x_3(t)$
$x_3(t) = \ddot{v}(t)$	$\dot{x}_3(t) = x_4(t)$
$x_4(t) = \dddot{v}(t)$	$\dot{x}_4(t) = -a_3x_4(t) - a_2x_3(t) - a_1x_2(t) - a_0x_1(t) + b_0u(t)$

(VI-5)

To convert the above into matrix form, with Equation (VI-5) in expanded form as:

$$\begin{aligned}
\dot{x}_1(t) &= 1\cdot x_2(t) \\
\dot{x}_2(t) &= 1\cdot x_3(t) \\
\dot{x}_3(t) &= 1\cdot x_4(t) \\
\dot{x}_4(t) &= -a_0x_1(t) - a_1x_2(t) - a_2x_3(t) - a_3x_4(t) + b_0u(t)
\end{aligned} \qquad \text{(VI-6)}$$

Example 1 (continued)

For clarity, it should be noted that the general vector equation represents a system of equations of the form:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \vdots \\ \dot{x}_n(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ \vdots \\ x_n(t) \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & \cdots & b_{1n} \\ b_{21} & b_{22} & b_{23} & \cdots & b_{2n} \\ b_{31} & b_{32} & b_{33} & \cdots & b_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & b_{n3} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ \vdots \\ u_n(t) \end{bmatrix} \quad \text{(VI-7)}$$

that have to be solved.

From Equation (VI-7) it readily follows that:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 & -a_3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ u(t) \end{bmatrix}$$

which may be written in vector form as:

$$\dot{\underline{x}}(t) = \underline{A}\,\underline{x} + \underline{B}\,\underline{u} \quad \text{(VI-8)}$$

Example 1 (continued)

The transfer function in terms of state-variables may easily be obtained by taking the Laplace transform of Equations (IV-3a) and (VI-3b).

$$\mathcal{L}\{\dot{\underline{X}} = \underline{A}\underline{X} + \underline{B}\underline{U}\} \qquad \text{(VI-8a)}$$

$$\mathcal{L}\{\underline{Y} = \underline{C}\underline{X} + \underline{D}\underline{U}\} \qquad \text{(VI-8b)}$$

resulting in

$$s\underline{X}(s) - \underline{X}(o) = \underline{A}\,\underline{X}(s) + \underline{B}\,\underline{U}(s) \qquad \text{(VI-9a)}$$

$$\underline{Y}(s) = \underline{C}\,\underline{X}(s) + \underline{D}\,\underline{U}(s) \qquad \text{(VI-9b)}$$

manipulating equation (VI-9a) in to a form readily substituted into (VI-9b)

$$(s\underline{I} - \underline{A})\,\underline{X}(s) = \underline{B}\,\underline{U}(s) + \underline{X}(o) \qquad \text{(VI-10)}$$

or

$$\underline{X}(s) = (s\underline{I} - \underline{A})^{-1}\,\underline{B}\,\underline{U}(s) + (s\underline{I} - \underline{A})^{-1}\,\underline{X}(o) \qquad \text{(VI-11)}$$

Therefore equation (VI-9b) becomes

$$\underline{Y}(s) = \underline{C}(s\underline{I} - \underline{A})^{-1}\,\underline{B}\,\underline{U}(s) + \underline{C}(s\underline{I} - A)^{-1}\,\underline{X}(o) + \underline{D}\,\underline{U}(s) \qquad \text{(VI-12)}$$

Example 1 (continued)

or

$$\underline{Y}(s) = \left(\underline{C}(s\underline{I}-\underline{A})^{-1}\underline{B} + \underline{D}\right)\underline{U}(s) + \underline{C}(s\underline{I}-\underline{A})^{-1}\underline{X}(0) \qquad \text{(VI-13)}$$

For the frequency domain considerations, the initial conditions are neglected, i.e., $\underline{X}(0) = 0$, therefore the transfer function in state-variable form is

$$\underline{T}(s) = \frac{\underline{Y}(s)}{\underline{U}(s)} = \underline{D} + \underline{C}(s\underline{I}-\underline{A})^{-1}\underline{B} \qquad \text{(VI-14)}$$

B. CONSIDERATIONS ON CONTROL TOPOLOGIES

Three simple examples will be considered, demonstrating the method of converting from network topology to a control topology.

Example 2:

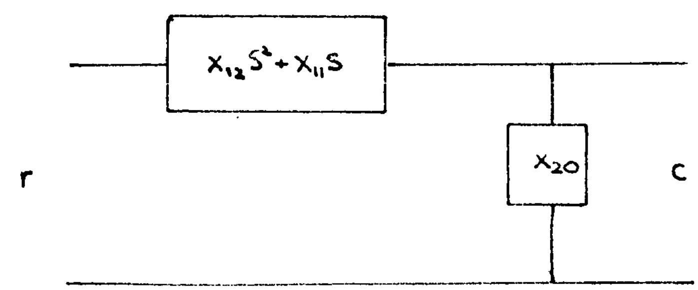


FIGURE VI-2. Network diagram of system #1

Example 2 (continued)

$$T(s) = \frac{C(s)}{R(s)} = \frac{X_{20}}{X_{12}S^2 + X_{11}S + X_{20}} \tag{VI-15}$$

$$= \frac{\dfrac{X_{20}/X_{12}}{S(S+X_{11}/X_{12})}}{1 + \dfrac{X_{20}/X_{12}}{S(S+X_{11}/X_{12})}}$$

The block diagram for the above equation is shown in Figure (VI-3)

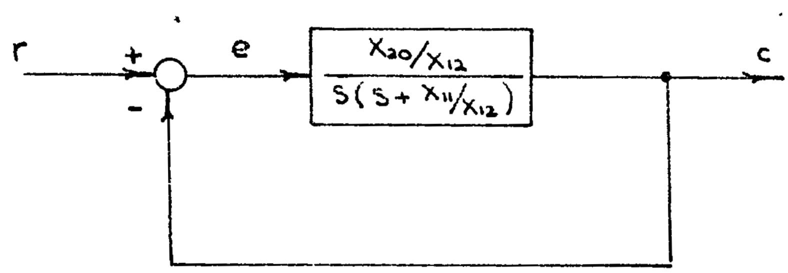


FIGURE VI-3. Control block diagram of system #1

Figure VI-3 may be easily redrawn and the state-variables defined as shown in Figure VI-4. (Note: the output of the integrators denote the state-variables x_p).

Example 2 (continued)

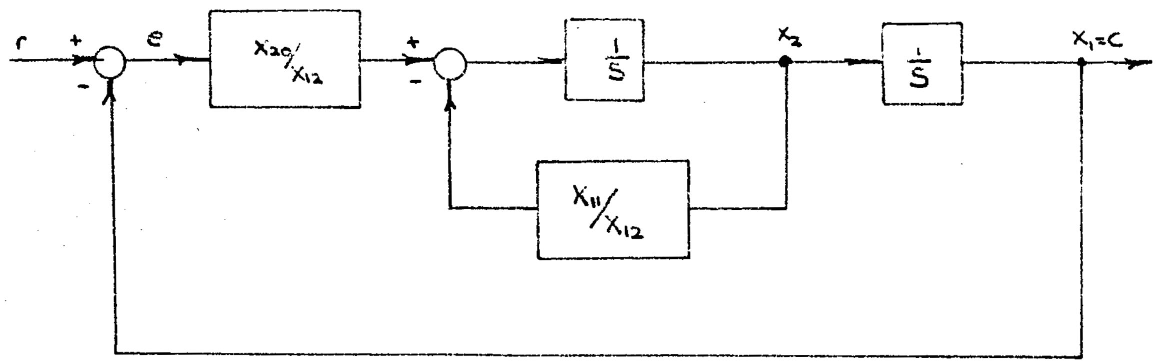


FIGURE VI-4. Control system #1 block diagram defined in terms of state-variables.

Example 3:

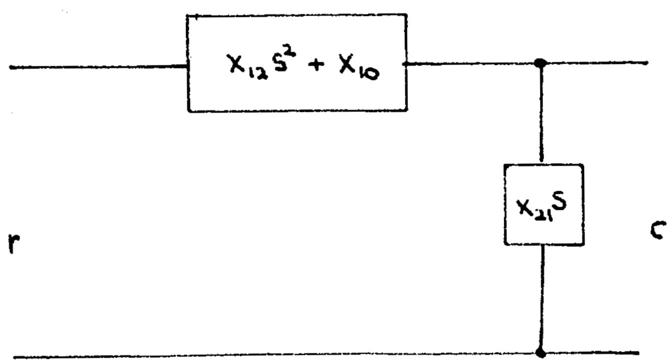


FIGURE VI-5. Network diagram of system #2

Example 3 (continued)

$$T(s) = \frac{C(s)}{R(s)} = \frac{X_{21}S}{X_{12}S^2 + X_{21}S + X_{10}} \tag{VI-16}$$

$$= X_{21}/X_{12} \; \frac{\dfrac{1}{S + X_{21}/X_{12}}}{1 + \dfrac{1}{(S + X_{21}/X_{12})} \times \dfrac{X_{10}/X_{12}}{S}}$$

The block diagram for the above equation is shown in Figure VI-6.

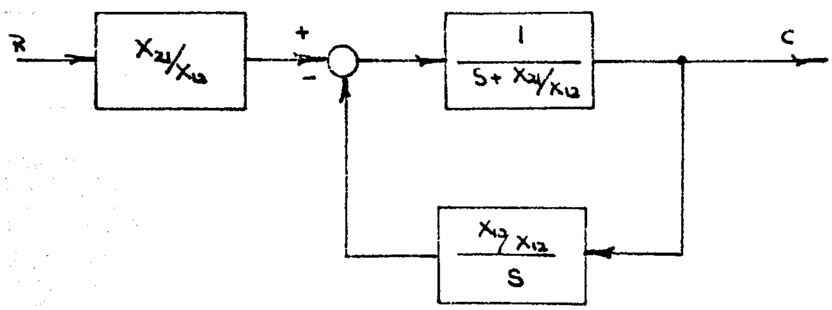


FIGURE VI-6. Control block diagram of system #2

Figure VI-6 may easily be redrawn and the state-variable defined as shown in Figure VI-7.

Example 3 (continued)

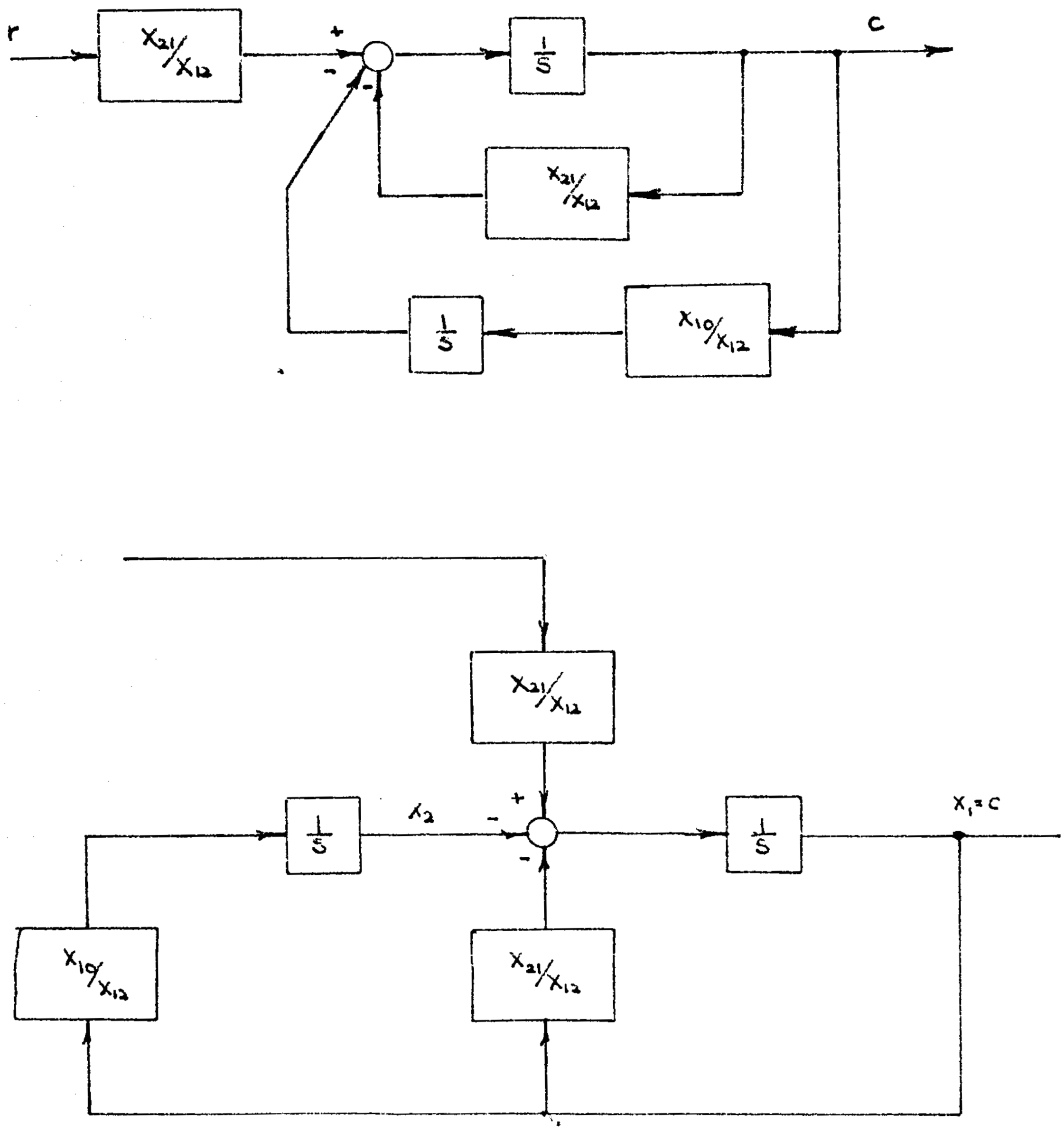


FIGURE VI-7. Control system #2 block diagram defined in terms of state-variables.

Example 4:

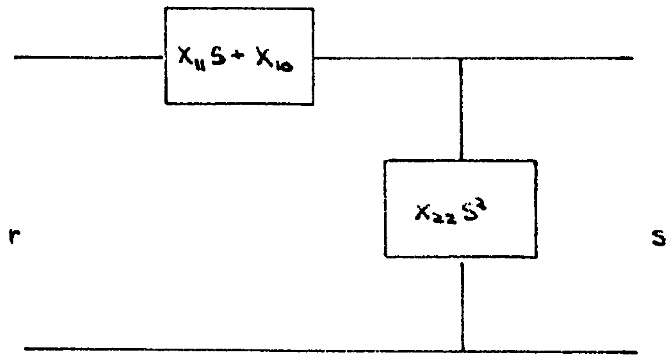


FIGURE VI-8. Network diagram of system #3

$$T(S) = \frac{C(S)}{R(S)} = \frac{X_{22}S^2}{X_{22}S^2 + X_{11}S + X_{10}}$$

$$= 1 - \left[\frac{\dfrac{X_{11}/X_{22}}{S + X_{11}/X_{22}}}{1 + \dfrac{X_{10}/X_{22}}{S(S + X_{11}/X_{22})}} + \frac{\dfrac{X_{10}/X_{22}}{S(S + X_{11}/X_{22})}}{1 + \dfrac{X_{10}/X_{22}}{S(S + X_{11}/X_{22})}} \right] \tag{VI-17}$$

Example 4 (continued)

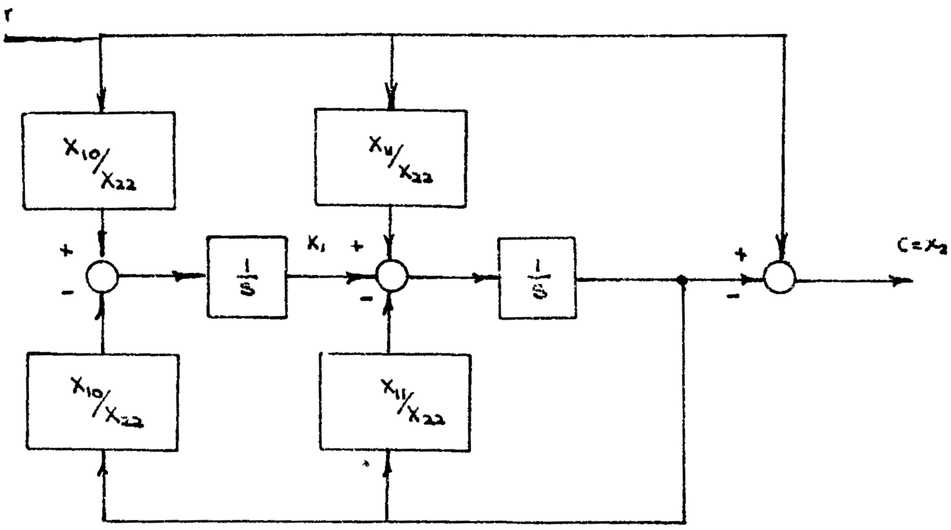


FIGURE VI-9. Control system #3 block diagram

A generalization of the control topology that can be used to represent the poles and zeros of a system defined by the transfer function

$$T(s) = \frac{\sum_{i=0}^{n} a_i s^i}{\sum_{j=0}^{m} b_j s^j} \qquad \text{(VI-18)}$$

is shown in Figure VI-10.

Example 4 (continued)

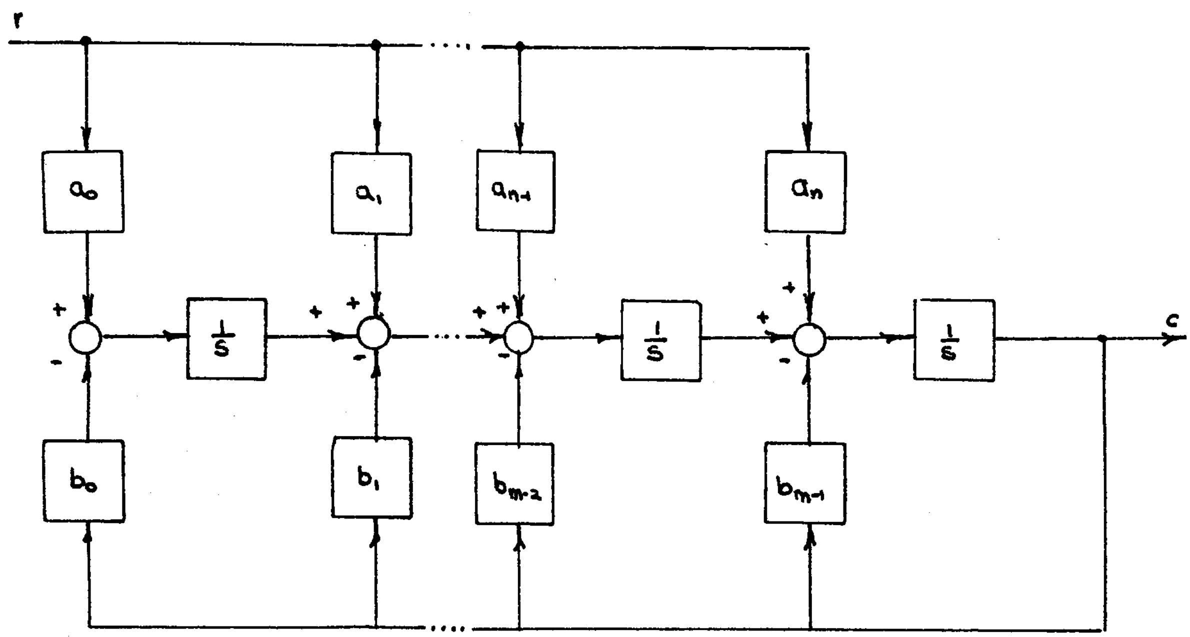


FIGURE VI-10. Generalized control system topology

SECTION VII

CONCLUSIONS AND RECOMMENDATIONS

A method for searching through a particular defined ordered space for that subset of systems satisfying the given requirements within that space has been presented. It has resulted in the definition of a generalized technique for synthesizing systems using network concepts to structure the problem and the digital computer for calculating the element values.

The concept of synthesizing systems from graph or topological considerations was developed in Sections II, III and IV using a quadratic impedance form to define the elements. This procedure allows for the determination of systems consisting of non-ideal passive elements, i.e., physically real systems as compared with the idealized elements determined by classical synthesis techniques. This concept is a direct approach to the optimization problem for it requires the enumeration of all possible systems within the ordered space satisfying the specifications. The optimum configuration, dependent upon the criteria, is then selected from among the calculated systems.

From the studies that have been completed it is apparent that further refinements are possible, starting with the machine expansion of the ordered space in a continuous fashion, enabling a progressive search for systems satisfying the requirements. The ultimate synthesis procedure as conceived in this report is one that is completely computer-mechanized from the input specifications to the output system topology having realistic component values. Consequently emphasis should be placed next on defining a programmable search mode for an expanded topology incorporating active elements.

SECTION VIII

REFERENCES

1. Firestone, F. A., "A New Analogy Between Mechanical Systems," J. Acoust. Soc. Am., 1933, pp. 248-267.

2. Olson, H. F., Dynamic Analogies (D. Van Nostrand, New York).

3. Kuh, E. S. and Rohrer, R. A., "The State Variable Approach," Proc. IEEE, Vol. 53, No. 7, July 1965, pp. 672-686.

4. Plunkett, R. (Editor), Colloquium on Mechanical Impedance Methods for Mechanical Vibrations, ASME Annual Meeting, New York, N.Y., December 2, 1958.

5. Szuchy, N., "Measurement of Non-Electrical Impedance," Paper, IEEE Instrumentation Subsection - Mach 1964.

6. Seshu, S. and Reed, M. B., Linear Graphs and Electrical Networks, (Reading, Mass., Addison-Wesley Publishing Co., Inc., 1961).